Completeness of Iris-based Program Logics

Zichen Zhang[*]

with

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A Missing Piece in Iris

Logic

Truth

A Missing Piece in Iris

Logic

Truth

$\mathsf{safe}_{\varphi}(e, \sigma)$

Never gets stuck

Result satisfies φ

A Missing Piece in Iris

$\vdash \mathsf{wp}\ e\ \{v.\ \ulcorner\varphi(v)\urcorner\}$

Logic

$\mathsf{safe}_{\varphi}(e, \sigma)$

Truth

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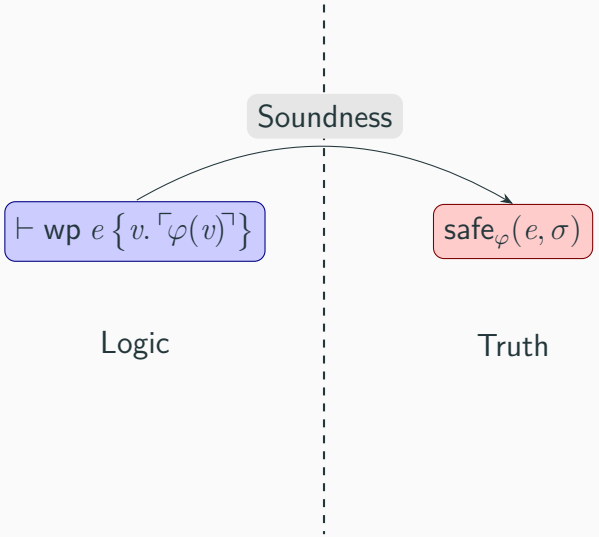
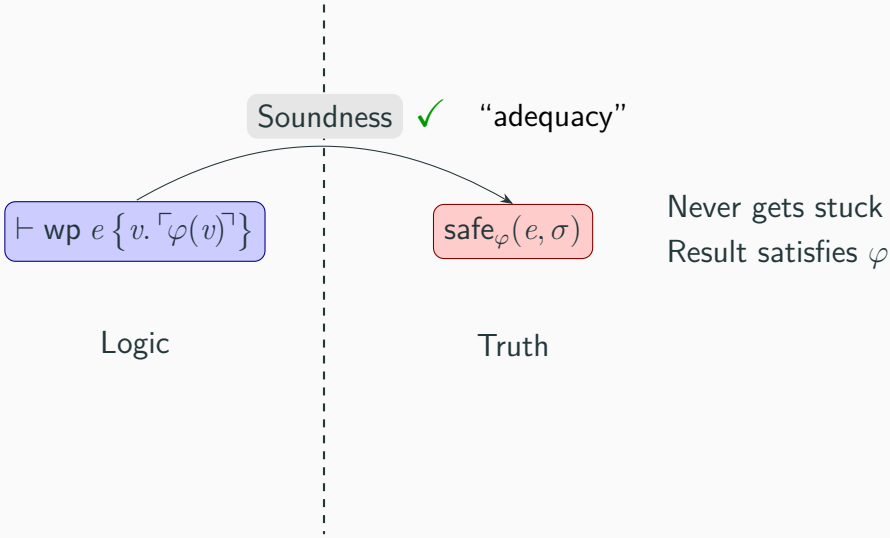
A Missing Piece in Iris



Never gets stuck

Result satisfies φ

A Missing Piece in Iris



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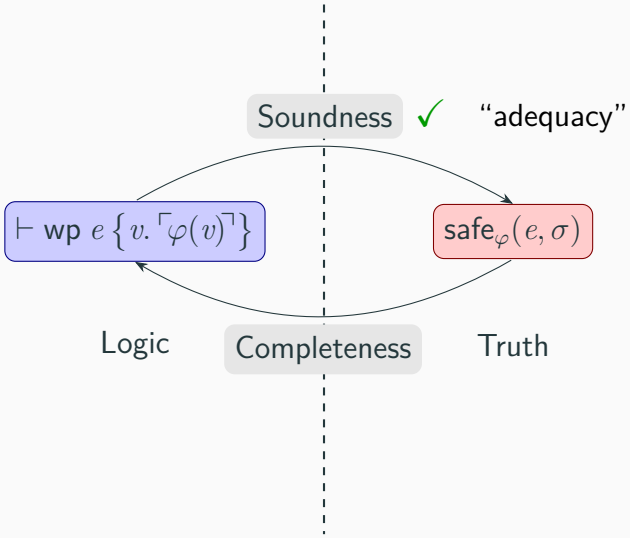
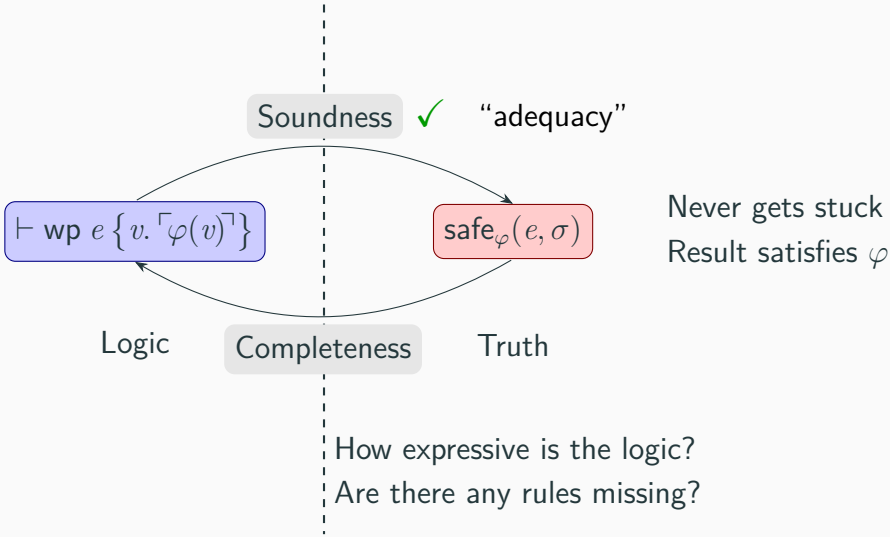
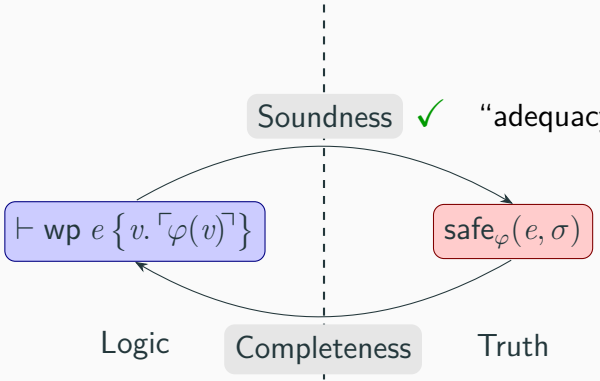


A Missing Piece in Iris



A Missing Piece in Iris

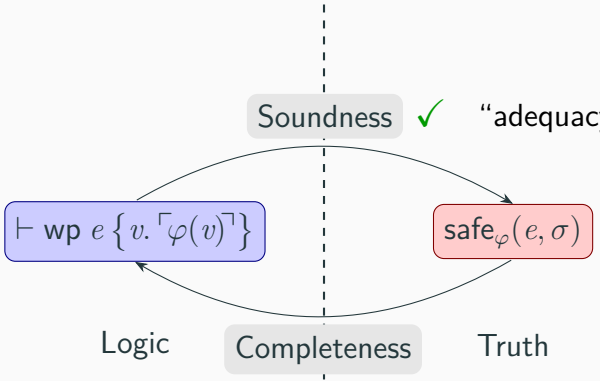


Prior work:

Cook (1978), Owicki (1975),
Jones (1983), Stirling (1988), etc.

How expressive is the logic?
Are there any rules missing?

A Missing Piece in Iris



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Cook (1978), Owicki (1975),
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Iris: ?

How expressive is the logic?

Are there any rules missing?

Our Contributions

Part 1: completeness of whole program specifications

- Generic completeness theorems for any Iris-based program logics for
  - Partial correctness
  - Total correctness
  - Refinement
- Concrete languages:
  - HeapLang, λ_{Rust}, Time credits, Eris
  - **Probably your favorite program logics!**

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Part 1: completeness of whole program specifications

- Generic completeness theorems for any Iris-based program logics for
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Part 2 (WIP): completeness of library specifications

- Completeness of logically atomic triples in Iris

Whole Program Completeness

Johannes Hostert **Zichen Zhang** Puming Liu

Simon Gregersen Ralf Jung Joseph Tassarotti

Warm-up: Sequential Completeness

Language: $\rightarrow_{\mathsf{pure}} \mid \texttt{ref}\ e \mid !\, e \mid e_1 \leftarrow e_2$

HeapLang, but without `fork`, `free`, and prophecy variables

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HeapLang, but without **`fork`**, **`free`**, and prophecy variables

$$\forall e \quad .\ \mathsf{safe}_{\varphi}(e, \varnothing) \implies \qquad \vdash \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}$$

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$$\forall e, \sigma.\, \mathsf{safe}_{\varphi}(e, \sigma) \implies \mathsf{S}_{\circ}(\sigma) \vdash \mathsf{wp}\ e \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}$$

$$\mathsf{S}_{\circ}(\sigma) \triangleq \mathop{\Large *}_{(\ell \leftarrow v) \in \sigma} (\ell \mapsto v) \qquad \text{(the ownership of } \sigma\text{)}$$

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Löb induction $\quad \triangleright \left(\forall e, \sigma.\, \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\} \right)$

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- If $\mathsf{safe}_\varphi(v, \sigma)$ ($v \in \mathit{Val}$)
- If $\mathsf{safe}_\varphi(e_1, \sigma_1)$ ($e_1 \notin \mathit{Val}$),

Warm-up: Sequential Completeness

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- If $\mathsf{safe}_\varphi(v, \sigma)$ ($v \in \mathit{Val}$) $\implies \ulcorner \varphi(v) \urcorner \vdash \mathsf{wp}\ v \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}$ (by WpValue)
- If $\mathsf{safe}_\varphi(e_1, \sigma_1)$ ($e_1 \notin \mathit{Val}$),

Warm-up: Sequential Completeness

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- If $\mathsf{safe}_\varphi(v, \sigma)$ ($v \in \mathit{Val}$) $\implies \ulcorner \varphi(v) \urcorner \vdash \mathsf{wp}\ v \left\{ v. \ulcorner \varphi(v) \urcorner \right\}$ (by WpValue)
- If $\mathsf{safe}_\varphi(e_1, \sigma_1)$ ($e_1 \notin \mathit{Val}$), then e_1 is reducible

case analysis on the first step

If $\mathsf{safe}_\varphi(e_1, \sigma_1)$ ($e_1 \notin \mathit{Val}$)

Pure hyp $\quad$ `Hsafe1`: $\mathsf{safe}_\varphi(e_1, \sigma_1)$

Persistent hyp $\quad$ `IH`: $\triangleright \left(\forall e, \sigma.\, \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}\right)$ □

Spatial hyp $\quad$ `H`$\circ$: $\mathsf{S}_\circ(\sigma_1)$

Goal $\quad$ $\mathsf{wp}\quad e_1\ \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}$ ∗

If $\mathsf{safe}_\varphi(e_1, \sigma_1)$ ($e_1 \notin \mathit{Val}$)

Pure hyp — $\texttt{Hsafe1}: \mathsf{safe}_\varphi(e_1, \sigma_1)$

Persistent hyp — $\texttt{IH}: \triangleright \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp — $\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

Goal — $\mathsf{wp}\ \ e_1\ \left\{ v. \ulcorner \varphi(v) \urcorner \right\}$

If $\mathsf{safe}_\varphi(e_1, \sigma_1)$ ($e_1 \notin \mathit{Val}$)

Pure hyp — $\texttt{Hsafe1}: \mathsf{safe}_\varphi(e_1, \sigma_1)$

$\texttt{Hprim}: (e_1, \sigma_1) \rightarrow_{\mathsf{prim}} (e_2, \sigma_2)$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(e_2, \sigma_2)$

Persistent hyp — $\texttt{IH}: \triangleright \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

---□

Spatial hyp — $\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

---∗

Goal — $\mathsf{wp}\quad e_1\ \left\{ v. \ulcorner \varphi(v) \urcorner \right\}$

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Spatial hyp — $\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

Goal — $\mathsf{wp}\quad e_1\ \left\{ v. \ulcorner \varphi(v) \urcorner \right\}$ ∗

If $\mathsf{safe}_\varphi(e_1, \sigma_1)$ ($e_1 \notin \mathit{Val}$)

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(e_1, \sigma_1)$

unfold prim-step $e_1 = K[e_1'] \land e_2 = K[e_2']$

$\texttt{Hprim}: (e_1, \sigma_1) \to_{\mathsf{prim}} (e_2, \sigma_2)$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(e_2, \sigma_2)$

Persistent hyp

$\texttt{IH}: \triangleright \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

---□

Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

---∗

Goal

$\mathsf{wp}\quad e_1 \left\{ v. \ulcorner \varphi(v) \urcorner \right\}$

If $\mathsf{safe}_\varphi(e_1, \sigma_1)$ ($e_1 \notin \mathit{Val}$)

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[e_1'], \sigma_1)$ unfold prim-step $e_1 = K[e_1'] \land e_2 = K[e_2']$

$\texttt{Hbase}: (e_1', \sigma_1) \to_{\mathsf{base}} (e_2', \sigma_2)$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[e_2'], \sigma_2)$

Persistent hyp $\texttt{IH}: \triangleright \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp $\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

Goal $\mathsf{wp}\ K[e_1'] \left\{ v. \ulcorner \varphi(v) \urcorner \right\}$

If $\mathsf{safe}_\varphi(e_1, \sigma_1)$ ($e_1 \notin \mathit{Val}$)

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[e_1'], \sigma_1)$

$\texttt{Hbase}: (e_1', \sigma_1) \rightarrow_{\mathsf{base}} (e_2', \sigma_2)$

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Persistent hyp

$\texttt{IH}: \triangleright \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

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$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

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Goal

$\mathsf{wp}\ K[e_1'] \left\{ v. \ulcorner \varphi(v) \urcorner \right\}$

If $\mathsf{safe}_\varphi(e_1, \sigma_1)$ ($e_1 \notin \mathit{Val}$)

Pure hyp

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Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

Goal

$\mathsf{wp}\ K[e_1'] \left\{ v. \ulcorner \varphi(v) \urcorner \right\}$ by WpBind

If $\mathsf{safe}_\varphi(e_1, \sigma_1)$ ($e_1 \notin \mathit{Val}$)

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[e_1'], \sigma_1)$

$\texttt{Hbase}: (e_1', \sigma_1) \rightarrow_{\mathsf{base}} (e_2', \sigma_2)$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[e_2'], \sigma_2)$

Persistent hyp

$\texttt{IH}: \triangleright \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

Goal

$\mathsf{wp}\ e_1' \left\{ w. \mathsf{wp}\ K[w] \left\{ v. \ulcorner \varphi(v) \urcorner \right\} \right\}$ by WpBind

If $\mathsf{safe}_\varphi(e_1, \sigma_1)$ ($e_1 \notin \mathit{Val}$)

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[e_1'], \sigma_1)$

$\texttt{Hbase}: (e_1', \sigma_1) \rightarrow_{\mathsf{base}} (e_2', \sigma_2)$

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Persistent hyp

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Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

Goal

$\mathsf{wp}\ e_1' \left\{ w. \mathsf{wp}\ K[w] \left\{ v. \ulcorner \varphi(v) \urcorner \right\} \right\}$

If $\mathsf{safe}_\varphi(e_1, \sigma_1)$ ($e_1 \notin \mathit{Val}$)

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[e_1'], \sigma_1)$ case analysis

$\texttt{Hbase}: (e_1', \sigma_1) \rightarrow_{\mathsf{base}} (e_2', \sigma_2)$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[e_2'], \sigma_2)$

Persistent hyp $\quad \texttt{IH}: \triangleright \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp $\quad \texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

Goal $\quad \mathsf{wp}\ e_1' \left\{ w. \mathsf{wp}\ K[w] \left\{ v. \ulcorner \varphi(v) \urcorner \right\} \right\}$

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Persiste

Spatial

- $\square\ e_1' \rightarrow_{\mathsf{pure}} e_2'$
- $\square\ \sigma_1(\ell) = v \land (!\ell, \sigma_1) \rightarrow_{\mathsf{base}} (v, \sigma_1)$
- $\square\ \sigma_1(\ell) = v \land (\ell \leftarrow w, \sigma_1) \rightarrow_{\mathsf{base}} ((), \sigma_1[\ell \leftarrow w])$
- $\square\ \ell_0 \notin \sigma_1 \land (\texttt{ref}\ v, \sigma_1) \rightarrow_{\mathsf{base}} (\ell_0, \sigma_1[\ell_0 \leftarrow v])$

Goal

If e_1' takes a pure step

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[e_1'], \sigma_1)$

$\texttt{Hpure}: e_1' \to_{\mathsf{pure}} e_2'$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[e_2'], \sigma_1)$

Persistent hyp

$\texttt{IH}: \triangleright \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

---□

Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

---∗

Goal

$\mathsf{wp}\ e_1' \left\{ w. \mathsf{wp}\ K[w] \left\{ v. \ulcorner \varphi(v) \urcorner \right\} \right\}$

If e_1' takes a pure step

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[e_1'], \sigma_1)$

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Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

Goal

$\mathsf{wp}\ e_1' \left\{ w. \mathsf{wp}\ K[w] \left\{ v. \ulcorner \varphi(v) \urcorner \right\} \right\}$

by WpPure

If e_1' takes a pure step

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[e_1'], \sigma_1)$

$\texttt{Hpure}: e_1' \to_{\mathsf{pure}} e_2'$

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Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

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Goal

$\triangleright\, \mathsf{wp}\ e_2' \left\{ w.\ \mathsf{wp}\ K[w] \left\{ v. \ulcorner \varphi(v) \urcorner \right\} \right\}$

by WpPure

If e_1' takes a pure step

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$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[e_1'], \sigma_1)$

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$\texttt{IH}: \triangleright \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

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Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

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Goal

$\triangleright\, \mathsf{wp}\ e_2' \left\{ w.\ \mathsf{wp}\ K[w] \left\{ v. \ulcorner \varphi(v) \urcorner \right\} \right\}$

If e_1' takes a pure step

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[e_1'], \sigma_1)$

$\texttt{Hpure}: e_1' \to_{\mathsf{pure}} e_2'$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[e_2'], \sigma_1)$

Persistent hyp

$\texttt{IH}: \triangleright \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

strip later

Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

Goal

$\triangleright\, \mathsf{wp}\ e_2' \left\{ w.\ \mathsf{wp}\ K[w] \left\{ v. \ulcorner \varphi(v) \urcorner \right\} \right\}$

If e_1' takes a pure step

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[e_1'], \sigma_1)$

$\texttt{Hpure}: e_1' \to_{\mathsf{pure}} e_2'$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[e_2'], \sigma_1)$

Persistent hyp

$\texttt{IH}: \quad (\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \{ v. \ulcorner \varphi(v) \urcorner \})$

strip later □

Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

Goal

$\mathsf{wp}\ e_2' \{ w.\ \mathsf{wp}\ K[w] \{ v. \ulcorner \varphi(v) \urcorner \} \}$ ∗

If e_1' takes a pure step

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[e_1'], \sigma_1)$

$\texttt{Hpure}: e_1' \to_{\mathsf{pure}} e_2'$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[e_2'], \sigma_1)$

Persistent hyp

$\texttt{IH}: \quad \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

---□

Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

---∗

Goal

$\mathsf{wp}\ e_2' \left\{ w.\ \mathsf{wp}\ K[w] \left\{ v. \ulcorner \varphi(v) \urcorner \right\} \right\}$

If e_1' takes a pure step

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[e_1'], \sigma_1)$

$\texttt{Hpure}: e_1' \to_{\mathsf{pure}} e_2'$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[e_2'], \sigma_1)$

Persistent hyp

$\texttt{IH}: \quad \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

Goal

$\mathsf{wp}\ e_2' \left\{ w.\ \mathsf{wp}\ K[w] \left\{ v. \ulcorner \varphi(v) \urcorner \right\} \right\}$

by WpBindInv

If e_1' takes a pure step

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[e_1'], \sigma_1)$

$\texttt{Hpure}: e_1' \to_{\mathsf{pure}} e_2'$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[e_2'], \sigma_1)$

Persistent hyp

$\texttt{IH}: \quad \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

---□

Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

---✱

Goal

$\mathsf{wp}\ K[e_2'] \left\{ v. \ulcorner \varphi(v) \urcorner \right\}$

by WpBindInv

If e_1' takes a pure step

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[e_1'], \sigma_1)$

$\texttt{Hpure}: e_1' \to_{\mathsf{pure}} e_2'$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[e_2'], \sigma_1)$

Persistent hyp

$\texttt{IH}: \quad \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

---□

Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

---∗

Goal

$\mathsf{wp}\ K[e_2'] \left\{ v. \ulcorner \varphi(v) \urcorner \right\}$

If e_1' takes a pure step

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[e_1'], \sigma_1)$

$\texttt{Hpure}: e_1' \to_{\mathsf{pure}} e_2'$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[e_2'], \sigma_1)$

Persistent hyp

$\texttt{IH}: \quad \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$ apply

---

Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

Goal

$\mathsf{wp}\ K[e_2'] \left\{ v. \ulcorner \varphi(v) \urcorner \right\}$

If e_1' takes a pure step

Pure hyp `Hsafe1`: $\mathsf{safe}_\varphi(K[e_1'], \sigma_1)$

`Hpure`: $e_1' \to_{\mathsf{pure}} e_2'$

`Hsafe2`: $\mathsf{safe}_\varphi(K[e_2'], \sigma_1)$

Persistent hyp `IH`: $\left(\forall e, \sigma.\ \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v.\ \ulcorner \varphi(v) \urcorner \right\}\right)$ apply

---

Spatial hyp `Ho`: $\mathsf{S}_\circ(\sigma_1)$

Goals $\ulcorner \mathsf{safe}_\varphi(K[e_2'], \sigma_1) \urcorner$ $\quad$ $\mathsf{S}_\circ(\sigma_1)$

If e_1' takes a pure step

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[e_1'], \sigma_1)$

$\texttt{Hpure}: e_1' \to_{\mathsf{pure}} e_2'$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[e_2'], \sigma_1)$ frame

---

Persistent hyp

$\texttt{IH}: \quad \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$ frame

Goals

$\ulcorner \mathsf{safe}_\varphi(K[e_2'], \sigma_1) \urcorner$ $\mathsf{S}_\circ(\sigma_1)$

If e_1' takes a pure step

Pure hyp `Hsafe1`: $\mathsf{safe}_\varphi(K[e_1'], \sigma_1)$

- ☑ $e_1' \to_{\mathsf{pure}} e_2'$
- ☐ $\sigma_1(\ell) = v \land (!\ell, \sigma_1) \to_{\mathsf{base}} (v, \sigma_1)$
- ☐ $\sigma_1(\ell) = v \land (\ell \leftarrow w, \sigma_1) \to_{\mathsf{base}} ((), \sigma_1[\ell \leftarrow w])$
- ☐ $\ell_0 \notin \sigma_1 \land (\mathbf{ref}\ v, \sigma_1) \to_{\mathsf{base}} (\ell_0, \sigma_1[\ell_0 \leftarrow v])$

Persiste

Spatial

Goal ✓

If $e_1' = !\,\ell$

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[!\,\ell], \sigma_1)$ $\qquad$ $\texttt{Hlookup}: \sigma_1(\ell) = v$

$\texttt{Hbase}: (!\,\ell, \sigma_1) \rightarrow_{\mathsf{base}} (v, \sigma_1)$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[v], \sigma_1)$

Persistent hyp

$\texttt{IH}: \triangleright \left(\forall e, \sigma.\, \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

Goal

$\mathsf{wp}\ !\,\ell \left\{ w.\, \mathsf{wp}\ K[w] \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\} \right\}$

If $e_1' = !\,\ell$

Pure hyp

$\texttt{Hsafe1}\colon \mathsf{safe}_\varphi(K[!\,\ell], \sigma_1)$ $\qquad$ $\texttt{Hlookup}\colon \sigma_1(\ell) = v$

$\texttt{Hbase}\colon (!\,\ell, \sigma_1) \to_{\mathsf{base}} (v, \sigma_1)$

$\texttt{Hsafe2}\colon \mathsf{safe}_\varphi(K[v], \sigma_1)$

Persistent hyp $\quad \texttt{IH}\colon \triangleright \left(\forall e, \sigma.\, \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp $\quad \texttt{H}\circ\colon \mathsf{S}_\circ(\sigma_1)$ $\qquad$ destruct $\qquad \mathsf{S}_\circ(\sigma_1) \triangleq \ast_{(\ell \leftarrow v) \in \sigma_1} (\ell \mapsto v)$

Goal $\quad \mathsf{wp}\ !\,\ell \left\{ w.\, \mathsf{wp}\ K[w] \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\} \right\}$

If $e_1' = !\,\ell$

Pure hyp

Hsafe1: $\mathsf{safe}_\varphi(K[!\,\ell], \sigma_1)$

Hlookup: $\sigma_1(\ell) = v$

Hbase: $(!\,\ell, \sigma_1) \rightarrow_{\mathsf{base}} (v, \sigma_1)$

Hsafe2: $\mathsf{safe}_\varphi(K[v], \sigma_1)$

Persistent hyp

IH: $\triangleright \left(\forall e, \sigma.\ \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v.\ \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp

Ho: $\mathsf{S}_\circ(\sigma_1 \setminus \{\ell\})$

Hl: $\ell \mapsto v$

destruct $\mathsf{S}_\circ(\sigma_1) \triangleq \ast_{(\ell \leftarrow v) \in \sigma_1} (\ell \mapsto v)$

Goal

$\mathsf{wp}\ !\,\ell \left\{ w.\ \mathsf{wp}\ K[w] \left\{ v.\ \ulcorner \varphi(v) \urcorner \right\} \right\}$

If $e_1' = !\,\ell$

Pure hyp

$\texttt{Hsafe1}\colon \mathsf{safe}_\varphi(K[!\,\ell], \sigma_1)$ $\qquad$ $\texttt{Hlookup}\colon \sigma_1(\ell) = v$

$\texttt{Hbase}\colon (!\,\ell, \sigma_1) \rightarrow_{\mathsf{base}} (v, \sigma_1)$

$\texttt{Hsafe2}\colon \mathsf{safe}_\varphi(K[v], \sigma_1)$

Persistent hyp

$\texttt{IH}\colon \triangleright \left(\forall e, \sigma.\, \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp

$\texttt{H}\circ\colon \mathsf{S}_\circ(\sigma_1 \setminus \{\ell\})$

$\texttt{Hl}\colon \ell \mapsto v$

Goal

$\mathsf{wp}\ !\,\ell \left\{ w.\, \mathsf{wp}\ K[w] \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\} \right\}$

If $e_1' = !\,\ell$

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[!\,\ell], \sigma_1)$ $\qquad$ $\texttt{Hlookup}: \sigma_1(\ell) = v$

$\texttt{Hbase}: (!\,\ell, \sigma_1) \rightarrow_{\mathsf{base}} (v, \sigma_1)$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[v], \sigma_1)$

Persistent hyp $\quad \texttt{IH}: \triangleright \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1 \setminus \{\ell\})$

$\texttt{Hl}: \ell \mapsto v$

Goal $\quad \mathsf{wp}\ !\,\ell \left\{ w. \mathsf{wp}\ K[w] \left\{ v. \ulcorner \varphi(v) \urcorner \right\} \right\}$

by WpLoad

If $e_1' = !\,\ell$

Pure hyp

`Hsafe1`: $\mathsf{safe}_\varphi(K[!\,\ell], \sigma_1)$ `Hlookup`: $\sigma_1(\ell) = v$

`Hbase`: $(!\,\ell, \sigma_1) \rightarrow_{\mathsf{base}} (v, \sigma_1)$

`Hsafe2`: $\mathsf{safe}_\varphi(K[v], \sigma_1)$

Persistent hyp `IH`: $\triangleright\, \left(\forall e, \sigma.\, \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e\, \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp `H`$\circ$: $\mathsf{S}_\circ(\sigma_1 \setminus \{\ell\})$

Goal $\triangleright\, (\ell \mapsto v \mathrel{-\!\!*} \mathsf{wp}\ K[v]\, \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\})$

by WpLoad

If $e_1' = \,!\,\ell$

Pure hyp

$\texttt{Hsafe1}\colon \mathsf{safe}_\varphi(K[!\,\ell], \sigma_1)$ $\qquad$ $\texttt{Hlookup}\colon \sigma_1(\ell) = v$

$\texttt{Hbase}\colon (!\,\ell, \sigma_1) \rightarrow_{\mathsf{base}} (v, \sigma_1)$

$\texttt{Hsafe2}\colon \mathsf{safe}_\varphi(K[v], \sigma_1)$

Persistent hyp $\quad \texttt{IH}\colon \triangleright \left(\forall e, \sigma.\, \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp $\quad \texttt{H}\circ\colon \mathsf{S}_\circ(\sigma_1 \setminus \{\ell\})$

Goal $\quad \triangleright (\ell \mapsto v \mathrel{-\!\!*} \mathsf{wp}\ K[v] \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\})$

If $e_1' = !\,\ell$

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[!\,\ell], \sigma_1)$ $\quad$ $\texttt{Hlookup}: \sigma_1(\ell) = v$

$\texttt{Hbase}: (!\,\ell, \sigma_1) \rightarrow_{\mathsf{base}} (v, \sigma_1)$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[v], \sigma_1)$

Persistent hyp $\quad \texttt{IH}: \triangleright \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp $\quad \texttt{H}\circ: \mathsf{S}_\circ(\sigma_1 \setminus \{\ell\})$

Goal $\quad \triangleright (\ell \mapsto v \mathrel{-\!\!*} \mathsf{wp}\ K[v] \left\{ v. \ulcorner \varphi(v) \urcorner \right\})$

strip later and intro

If $e_1' = !\,\ell$

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[!\,\ell], \sigma_1)$ $\qquad$ $\texttt{Hlookup}: \sigma_1(\ell) = v$

$\texttt{Hbase}: (!\,\ell, \sigma_1) \rightarrow_{\mathsf{base}} (v, \sigma_1)$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[v], \sigma_1)$

Persistent hyp

$\texttt{IH}: \quad \left(\forall e, \sigma.\, \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1 \setminus \{\ell\})$

$\texttt{Hl}: \ell \mapsto v$

Goal

$\mathsf{wp}\ K[v] \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}$

strip later and intro

If $e_1' = !\,\ell$

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[!\,\ell], \sigma_1)$ $\qquad$ $\texttt{Hlookup}: \sigma_1(\ell) = v$

$\texttt{Hbase}: (!\,\ell, \sigma_1) \rightarrow_{\mathsf{base}} (v, \sigma_1)$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[v], \sigma_1)$

Persistent hyp

$\texttt{IH}: \quad \left(\forall e, \sigma.\, \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}\right)$ □

Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1 \setminus \{\ell\})$

$\texttt{Hl}: \ell \mapsto v$

Goal

$\mathsf{wp}\ K[v] \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}$ ∗

If $e_1' = !\,\ell$

Pure hyp

Hsafe1: $\mathsf{safe}_\varphi(K[!\,\ell], \sigma_1)$ Hlookup: $\sigma_1(\ell) = v$

Hbase: $(!\,\ell, \sigma_1) \to_{\mathsf{base}} (v, \sigma_1)$

Hsafe2: $\mathsf{safe}_\varphi(K[v], \sigma_1)$

Persistent hyp

IH: $\left(\forall e, \sigma.\ \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v.\ \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp

Ho: $\mathsf{S}_\circ(\sigma_1 \setminus \{\ell\})$

Hl: $\ell \mapsto v$

combine $\mathsf{S}_\circ(\sigma_1) \triangleq \ast_{(\ell \leftarrow v) \in \sigma_1} (\ell \mapsto v)$

Goal

$\mathsf{wp}\ K[v] \left\{ v.\ \ulcorner \varphi(v) \urcorner \right\}$

If $e_1' = !\,\ell$

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[!\,\ell], \sigma_1)$ $\qquad$ $\texttt{Hlookup}: \sigma_1(\ell) = v$

$\texttt{Hbase}: (!\,\ell, \sigma_1) \to_{\mathsf{base}} (v, \sigma_1)$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[v], \sigma_1)$

Persistent hyp $\quad \texttt{IH}: \quad \left(\forall e, \sigma.\, \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp $\quad \texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

combine $\quad \mathsf{S}_\circ(\sigma_1) \triangleq \mathop{\Large *}_{(\ell \leftarrow v) \in \sigma_1} (\ell \mapsto v)$

Goal $\quad \mathsf{wp}\ K[v] \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}$

If $e_1' = !\,\ell$

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[!\,\ell], \sigma_1)$ $\qquad$ $\texttt{Hlookup}: \sigma_1(\ell) = v$

$\texttt{Hbase}: (!\,\ell, \sigma_1) \rightarrow_{\mathsf{base}} (v, \sigma_1)$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[v], \sigma_1)$

Persistent hyp $\quad \texttt{IH}: \quad \left(\forall e, \sigma.\, \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp $\quad \texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

Goal $\quad \mathsf{wp}\ K[v] \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}$

If $e_1' = !\,\ell$

Pure hyp

`Hsafe1`: $\mathsf{safe}_\varphi(K[!\,\ell], \sigma_1)$ `Hlookup`: $\sigma_1(\ell) = v$

`Hbase`: $(!\,\ell, \sigma_1) \rightarrow_{\mathsf{base}} (v, \sigma_1)$

`Hsafe2`: $\mathsf{safe}_\varphi(K[v], \sigma_1)$ frame

Persistent hyp `IH`: $\left(\forall e, \sigma.\ \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e\ \left\{ v.\ \ulcorner \varphi(v) \urcorner \right\}\right)$ apply

Spatial hyp `H`$\circ$: $\mathsf{S}_\circ(\sigma_1)$ frame

Goal $\mathsf{wp}\ K[v]\ \left\{ v.\ \ulcorner \varphi(v) \urcorner \right\}$

If $e_1' = !\,\ell$

Pure hyp

`Hsafe1`: $\mathsf{safe}_\varphi(K[!\,\ell], \sigma_1)$ `Hlookup`: $\sigma_1(\ell) = v$

`Hbase`: $(!\,\ell, \sigma_1) \rightarrow_{\mathsf{base}} (v, \sigma_1)$

`Hsafe2`: $\mathsf{safe}_\varphi(K[v], \sigma_1)$

Persistent hyp `IH`: $\left(\forall e, \sigma.\, \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}\right)$ apply

---

Spatial hyp `H`$\circ$: $\mathsf{S}_\circ(\sigma_1)$ framed

Goal ✓

If $e_1' = !\,\ell$

Pure hyp $\quad$ `Hsafe1`: $\mathsf{safe}_\varphi(K[!\,\ell], \sigma_1)$ $\quad$ `Hlookup`: $\sigma_1(\ell) = v$

Persiste

Spatial

- ☑ $e_1' \to_{\mathsf{pure}} e_2'$
- ☑ $\sigma_1(\ell) = v \land (!\ell, \sigma_1) \to_{\mathsf{base}} (v, \sigma_1)$
- ☐ $\sigma_1(\ell) = v \land (\ell \leftarrow w, \sigma_1) \to_{\mathsf{base}} ((), \sigma_1[\ell \leftarrow w])$
- ☐ $\ell_0 \notin \sigma_1 \land (\mathbf{ref}\ v, \sigma_1) \to_{\mathsf{base}} (\ell_0, \sigma_1[\ell_0 \leftarrow v])$

Goal $\quad$ ✓

If $e_1' = \ell \leftarrow w$

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[\ell \leftarrow w], \sigma_1)$ $\quad$ $\texttt{Hlookup}: \sigma_1(\ell) = v$

$\texttt{Hbase}: (\ell \leftarrow w, \sigma_1) \rightarrow_{\mathsf{base}} ((), \sigma_1[\ell \leftarrow w])$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[()], \sigma_1[\ell \leftarrow w])$

Persistent hyp

$\texttt{IH}: \triangleright \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

---□

Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

---∗

Goal

$\mathsf{wp}\ \ell \leftarrow w \left\{ w. \mathsf{wp}\ K[w] \left\{ v. \ulcorner \varphi(v) \urcorner \right\} \right\}$

If $e_1' = \ell \leftarrow w$

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[\ell \leftarrow w], \sigma_1)$ $\quad$ $\texttt{Hlookup}: \sigma_1(\ell) = v$

$\texttt{Hbase}: (\ell \leftarrow w, \sigma_1) \rightarrow_{\mathsf{base}} ((), \sigma_1[\ell \leftarrow w])$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[()], \sigma_1[\ell \leftarrow w])$

Persistent hyp $\quad \texttt{IH}: \triangleright \left(\forall e, \sigma. \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp $\quad \texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

Goal $\quad \mathsf{wp}\ \ell \leftarrow w \left\{ w. \mathsf{wp}\ K[w] \left\{ v. \ulcorner \varphi(v) \urcorner \right\} \right\}$

by WpStore

If $e_1' = \ell \leftarrow w$

Pure hyp $\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[\ell \leftarrow w], \sigma_1)$ $\texttt{Hlookup}: \sigma_1(\ell) = v$

☑ $e_1' \to_{\mathsf{pure}} e_2'$

☑ $\sigma_1(\ell) = v \land (!\ell, \sigma_1) \to_{\mathsf{base}} (v, \sigma_1)$

Persiste ☑ $\sigma_1(\ell) = v \land (\ell \leftarrow w, \sigma_1) \to_{\mathsf{base}} ((), \sigma_1[\ell \leftarrow w])$

Spatial ☐ $\ell_0 \notin \sigma_1 \land (\texttt{ref}\ v, \sigma_1) \to_{\mathsf{base}} (\ell_0, \sigma_1[\ell_0 \leftarrow v])$

Goal $\mathsf{wp}\ \ell \leftarrow w \left\{ w.\ \mathsf{wp}\ K[w] \left\{ v.\ \ulcorner \varphi(v) \urcorner \right\} \right\}$ $*$

by WpStore

If $e_1' = \texttt{ref}\ v$

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[\texttt{ref}\ v], \sigma_1)$

$\texttt{Hbase}: (\texttt{ref}\ v, \sigma_1) \rightarrow_{\mathsf{base}} (\ell_0, \sigma_1[\ell_0 \leftarrow v])$

$\texttt{Hsafe2}: \mathsf{safe}_\varphi(K[\ell_0], \sigma_1[\ell_0 \leftarrow v])$

Persistent hyp

$\texttt{IH}: \triangleright \left(\forall e, \sigma.\ \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\circ(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v.\ \ulcorner \varphi(v) \urcorner \right\}\right)$

Spatial hyp

$\texttt{H}\circ: \mathsf{S}_\circ(\sigma_1)$

Goal

$\mathsf{wp}\ \texttt{ref}\ v \left\{ w.\ \mathsf{wp}\ K[w] \left\{ v.\ \ulcorner \varphi(v) \urcorner \right\} \right\}$

If $e_1' = \texttt{ref}\ v$

Pure hyp

$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[\texttt{ref}\ v], \sigma_1)$

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$\texttt{Hsafe1}: \mathsf{safe}_\varphi(K[\texttt{ref}\ v], \sigma_1)$ $\qquad$ $\texttt{Hfresh}: \ell \notin \sigma_1$

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ℓ_0 and ℓ are unrelated

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---□

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Spatial hyp

Goal $\quad \ulcorner \mathsf{safe}_\varphi(K[\ell], \sigma_1[\ell \leftarrow v]) \urcorner$

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pure intro

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$\Uparrow$

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$\mathsf{safe}_\varphi(K[\ell], \sigma_1[\ell \leftarrow v])$

$\Uparrow$

$(\texttt{ref}\ v, \sigma_1) \to_{\mathsf{base}} (\ell, \sigma_1[\ell \leftarrow v])$ ✓

(because **ref** chooses ℓ non-deterministically)

If $e_1' = \texttt{ref}\ v$

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$\mathsf{safe}_\varphi(K[\ell], \sigma_1[\ell \leftarrow v])$

$\Uparrow$

- ☑ $e_1' \to_{\mathsf{pure}} e_2'$
- ☑ $\sigma_1(\ell) = v \land (!\ell, \sigma_1) \to_{\mathsf{base}} (v, \sigma_1)$
- ☑ $\sigma_1(\ell) = v \land (\ell \leftarrow w, \sigma_1) \to_{\mathsf{base}} ((), \sigma_1[\ell \leftarrow w])$
- ☑ $\ell_0 \notin \sigma_1 \land (\texttt{ref}\ v, \sigma_1) \to_{\mathsf{base}} (\ell_0, \sigma_1[\ell_0 \leftarrow v])$

Recap

- Löb induction $\quad \triangleright \left(\forall e, \sigma.\, \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\diamond(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}\right)$
- If $\mathsf{safe}_\varphi(v, \sigma)$ ($v \in \mathit{Val}$) $\implies \ulcorner \varphi(v) \urcorner \vdash \mathsf{wp}\ v \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}$ (by $\textsc{WpValue}$)
- If $\mathsf{safe}_\varphi(e_1, \sigma_1)$ ($e_1 \notin \mathit{Val}$), then e_1 is reducible

  case analysis on the first step

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Language-independent, expect for the blue part

Recap

- Löb induction $\triangleright \left(\forall e, \sigma.\, \ulcorner \mathsf{safe}_\varphi(e, \sigma) \urcorner \mathrel{-\!\!*} \mathsf{S}_\diamond(\sigma) \mathrel{-\!\!*} \mathsf{wp}\ e \left\{ v.\, \ulcorner \varphi(v) \urcorner \right\}\right)$
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- If $\mathsf{safe}_\varphi(e_1, \sigma_1)$ ($e_1 \notin \mathit{Val}$), then e_1 is reducible

case analysis on the first step

Language-independent, expect for the blue part

For language-dependent part: update $\mathsf{S}_\diamond(\sigma)$ using primitive laws.

Concurrent Completeness

Sequential:

- Proof environment: $\mathsf{safe}_{\varphi}(e, \sigma)$ and $\mathsf{S}_{\circ}(\sigma)$
- One wp $e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}$

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But in concurrent setting, we need to prove...

$$\frac{\text{Spatial hyp} \qquad \ulcorner \mathsf{safe\text{-}tp}_\varphi(\vec{e}, \sigma) \urcorner \qquad \mathsf{S}_\circ(\sigma)}{\text{Goal} \qquad \mathsf{wp}\ \texttt{fork}\ \{e_1\}\,;\, e_2 \left\{ v. \ulcorner \varphi(v) \urcorner \right\}} *$$

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$$I_{compl} \triangleq \boxed{\exists \vec{e}, \sigma.\ \ulcorner\mathsf{safe\text{-}tp}_\varphi(\vec{e}, \sigma)\urcorner * \mathsf{S}_\circ(\sigma)}^{\mathcal{N}}$$

$$\forall i, e.\ I_{compl} \qquad \vdash \mathsf{wp}\ e\left\{v.\ulcorner i = 0 \Rightarrow \varphi(v)\urcorner\right\}$$

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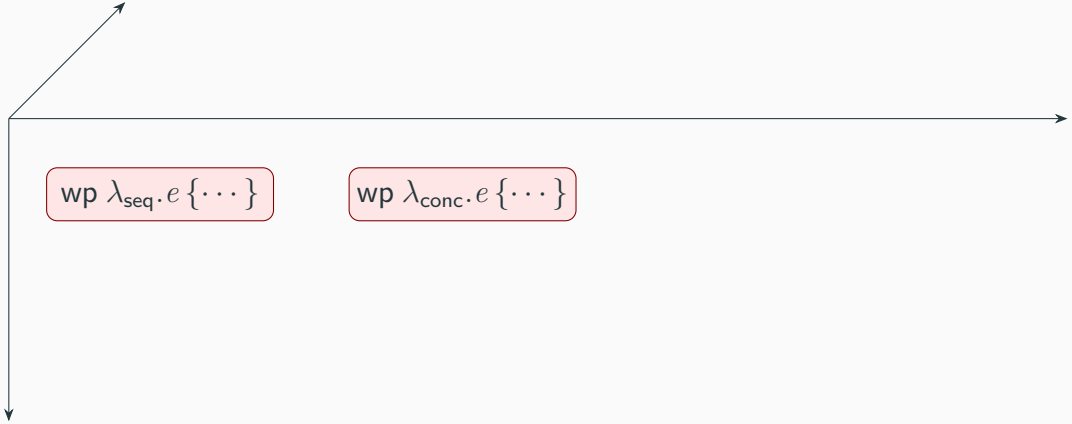
$$\forall i, e.\ I_{compl} * i \hookrightarrow^\gamma e \vdash \mathsf{wp}\ e\left\{v.\ulcorner i = 0 \Rightarrow \varphi(v)\urcorner\right\}$$

Generalization

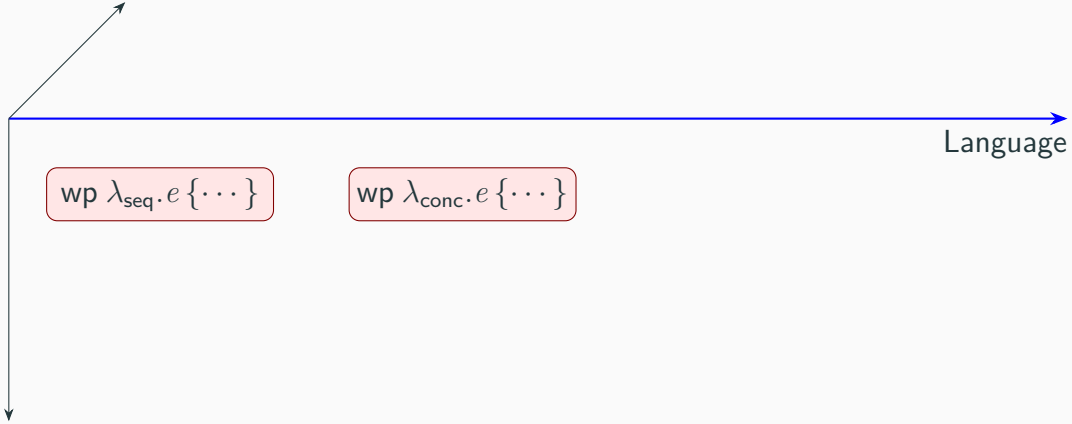
$\mathsf{wp}\ \lambda_{\mathsf{seq}}.e\,\{\cdots\}$

$\mathsf{wp}\ \lambda_{\mathsf{conc}}.e\,\{\cdots\}$

Generalization



Generalization



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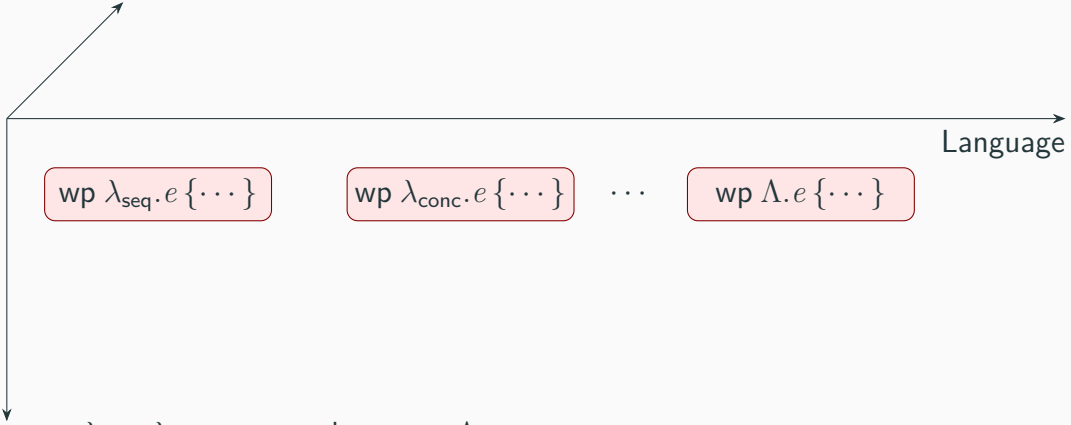
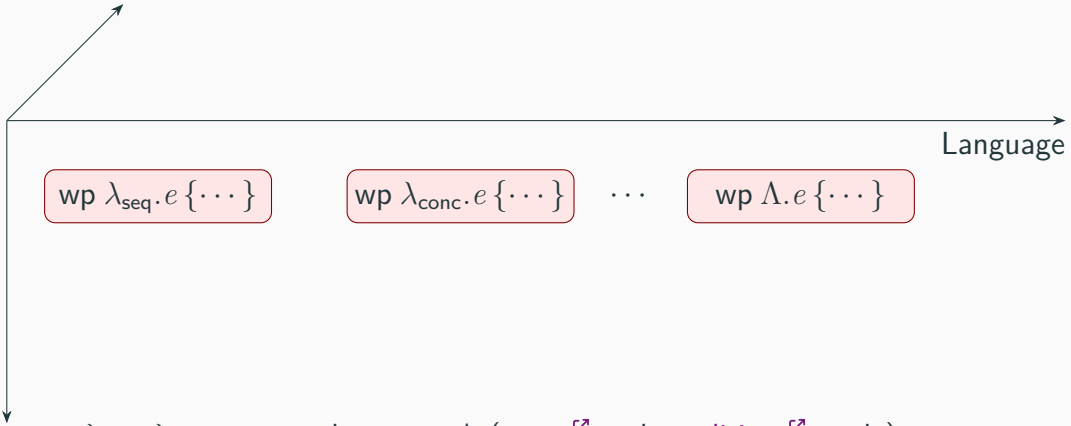


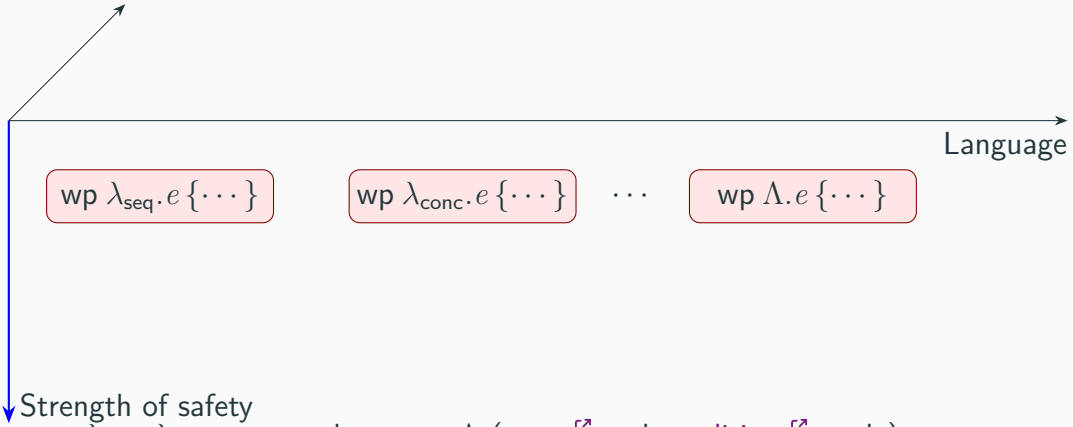
- λ_{seq}, $\lambda_{\mathsf{con}} \longrightarrow$ any language Λ

Generalization



- λ_{seq}, $\lambda_{\mathsf{con}} \longrightarrow$ any language Λ (terms and conditions apply)

Generalization



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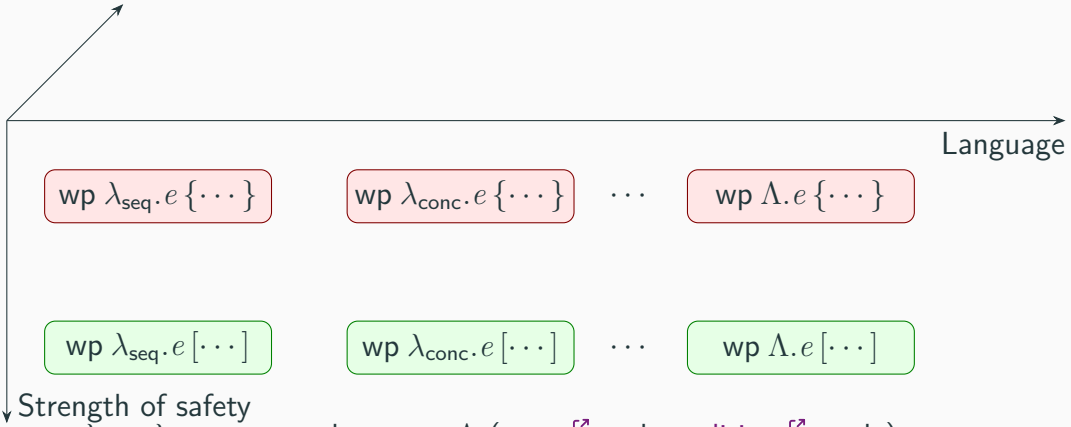

- λ_{seq}, $\lambda_{\text{con}} \longrightarrow$ any language Λ (terms and conditions apply)
- wp $e\,\{\cdots\} \longrightarrow$ wp $e\,[\cdots]$ (Löb induction $\longrightarrow$ well-founded induction)

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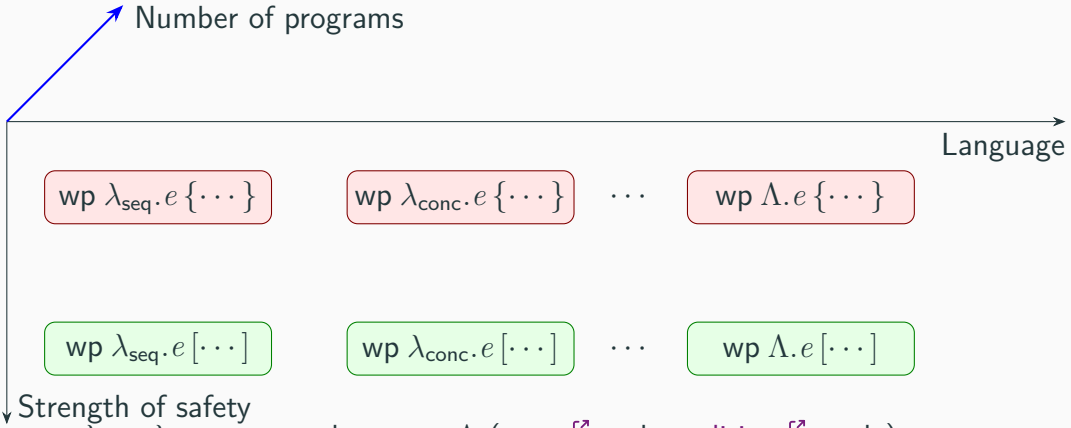

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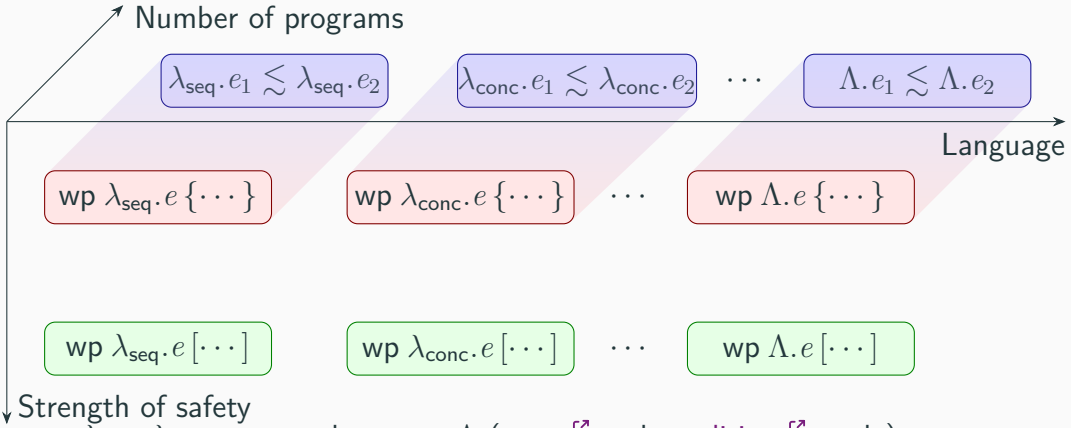

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- Unary $\longrightarrow$ binary (built on top of the unary completeness)

Condition 32 (Completeness).

$$\ulcorner \text{base-red}(e,\sigma)\urcorner * \mathrm{S}_\flat(\sigma) * \bullet^\gamma \vec{e} * \ulcorner \mathit{safe\text{-}tp}_\varphi(\vec{e},\sigma)\urcorner * n \hookrightarrow^\gamma K[e] \vdash \Rrightarrow_{\mathcal{E}}$$
$$\left(\ulcorner \text{Atomic } e\urcorner * \forall \Phi.\ \triangleright\ \mathit{WithStatePre}(\mathcal{E},e,n,K,\sigma,\vec{e},\Phi) \twoheadrightarrow \mathrm{wp}_{\mathcal{E}}\ e\ \{\Phi\}\right) \vee$$
$$\left(\mathrm{S}_\flat(\sigma) * \bullet^\gamma \vec{e} * \forall \Phi.\ \triangleright\ \mathit{WithoutStatePre}(\mathcal{E},e,K,\Phi) \twoheadrightarrow \mathrm{wp}_\top\ e\ \{\Phi\}\right)$$

where

$$\mathit{WithStatePre}(\mathcal{E},e,n,K,\sigma,\vec{e},\Phi) \triangleq \forall v',\sigma',\vec{e}_f.\ \ulcorner(e,\sigma) \to (v',\sigma',\vec{e}_f)\urcorner * \mathrm{S}_\flat(\sigma') * \bullet^\gamma \vec{e} * n \hookrightarrow^\gamma K[e] \twoheadrightarrow \Rrightarrow_{\mathcal{E}}$$
$$\Phi(v') * \mathop{\ast}_{e_f \in \vec{e}_f} \mathrm{wp}_\top\ e_f\ \{v.\ \mathrm{True}\}$$

$$\mathit{WithoutStatePre}(\mathcal{E},e,K,\Phi) \triangleq \forall e',\vec{e},\vec{e}_f.\ \mathit{Reduces}(K,e',\vec{e}) \twoheadrightarrow \Rrightarrow_\top$$
$$\mathrm{wp}_\top\ e'\ \{\Phi\} * \mathop{\ast}_{e_f \in \vec{e}_f} \mathrm{wp}_\top\ e_f\ \{v.\ \mathrm{True}\}$$

$$\mathit{Reduces}(e,K,e',\vec{e}) \triangleq \forall \sigma.\ \mathrm{S}_\flat(\sigma) * \bullet^\gamma \vec{e} * \ulcorner \mathit{safe\text{-}tp}_\varphi(\vec{e},\sigma)\urcorner \twoheadrightarrow \Rrightarrow_{\mathcal{E}}$$
$$\exists \sigma'.\ \ulcorner(e,\sigma) \to^+ (e',\sigma',\vec{e}_f)\urcorner * \mathrm{S}_\flat(\sigma') * \bullet^\gamma \vec{e} * n \hookrightarrow^\gamma K[e]$$

Completeness. If Condition 32 holds for a language Λ, then

$$\mathsf{safe}_\varphi(e,\varnothing) \implies \vdash \mathsf{wp}\ e\left\{v.\ \ulcorner\varphi(v)\urcorner\right\}$$

Strength of safety

- λ_{seq}, $\lambda_{\mathsf{con}} \longrightarrow$ any language Λ (terms and conditions apply)
- $\mathsf{wp}\ e\{\cdots\} \longrightarrow \mathsf{wp}\ e[\cdots]$ (Löb induction $\longrightarrow$ well-founded induction)
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- wp $e\{\cdots\}$ $\longrightarrow$ wp $e[\cdots]$ (Löb induction $\longrightarrow$ well-founded induction)
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Case Studies

Completeness. If Condition 32 holds for a language Λ, then

$$\mathsf{safe}_{\varphi}(e, \varnothing) \implies \vdash \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}$$

Case studies: HeapLang, λ_{Rust}, time credits, and Eris (error credits)

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Case studies: HeapLang, λ_{Rust}, time credits, and Eris (error credits)

- λ_{Rust}: handle non-atomic memory access

Case Studies

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Case studies: HeapLang, λ_{Rust}, time credits, and Eris (error credits)

- λ_{Rust}: handle non-atomic memory access
- Eris: redo the completeness theorem following the same pattern

The missing rules in HeapLang

$$
\begin{aligned}
\mathit{unique} \triangleq\ & \texttt{let}\ \ell_1 = \texttt{ref}()\ \texttt{in} \\
& \texttt{free}\ \ell_1; \\
& \texttt{assert}\ \ell_1 \neq \texttt{ref}()
\end{aligned}
$$

The missing rules in HeapLang

$$unique \triangleq \texttt{let}\ \ell_1 = \texttt{ref}()\ \texttt{in}\ \texttt{free}\ \ell_1;\ \texttt{assert}\ \ell_1 \neq \texttt{ref}()$$

Safe because locations are never reused!

$$\frac{\ell \notin \mathsf{dom}(\sigma)}{(\texttt{ref}\ v, \sigma) \rightarrow_{\mathsf{base}} (\ell, \sigma[\ell \leftarrow v], \epsilon)}$$

$$\frac{\sigma(\ell) = v}{(\texttt{free}\ \ell, \sigma) \rightarrow_{\mathsf{base}} ((), \sigma[\ell \leftarrow \bot]\,\sigma, \epsilon)}$$

The missing rules in HeapLang

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(was missing)

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Completeness.

For a HeapLang expression e, $\mathsf{safe}_{\varphi}(e, \varnothing) \implies \vdash \mathsf{wp}\ e \left\{ v. \ulcorner \varphi(v) \urcorner \right\}$

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Insight: What Iris features are needed?

- Löb induction
- First-order ghost state, ghost maps
- Invariants of timeless assertions
- WpValue, WpBind, WpPure, FupdWp, and WpAtomic

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We don't need

- Higher-order ghost state
- Impredicative invariants
- Later credits

Concurrent Library Completeness

Zichen Zhang Simon Gregersen Joseph Tassarotti

Logic & Truth for Concurrent Data Structures

Logic

Truth

Logic & Truth for Concurrent Data Structures

Logic

Truth

$\mathsf{lin}_\mu(\mathsf{op})$

Linearizable *w.r.t.* sequential behavior μ

Logic & Truth for Concurrent Data Structures

$I_\mu \vdash \langle P_\mu \rangle\ \mathsf{op}(obj, x)\ \langle y.\ Q_\mu(y) \rangle$

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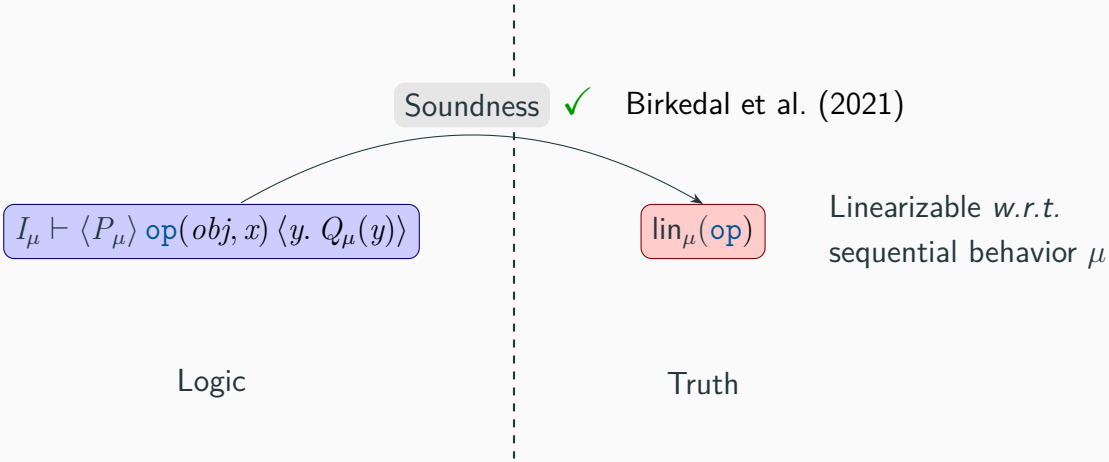
Logic & Truth for Concurrent Data Structures



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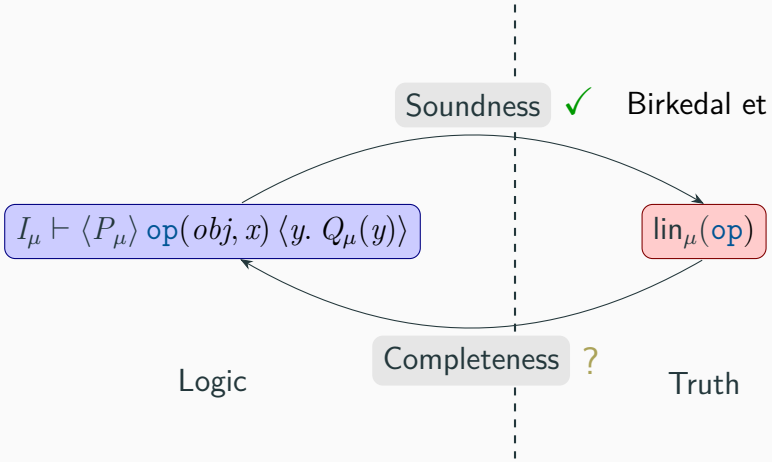
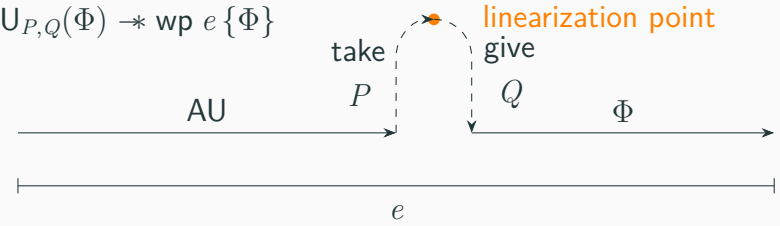


Key Ideas

$\langle P \rangle\ e\ \langle Q \rangle \triangleq \forall \Phi.\ \mathsf{AU}_{P,Q}(\Phi) \twoheadrightarrow \mathsf{wp}\ e\ \{\Phi\}$



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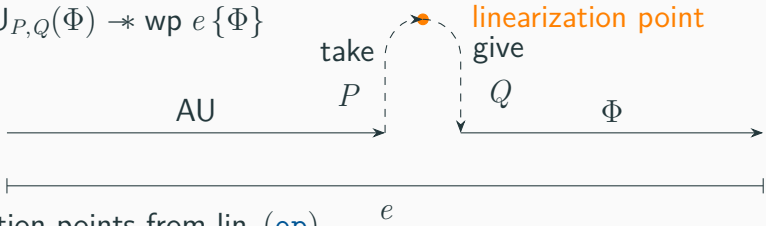

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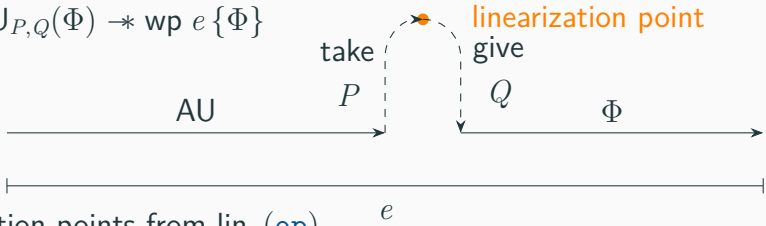

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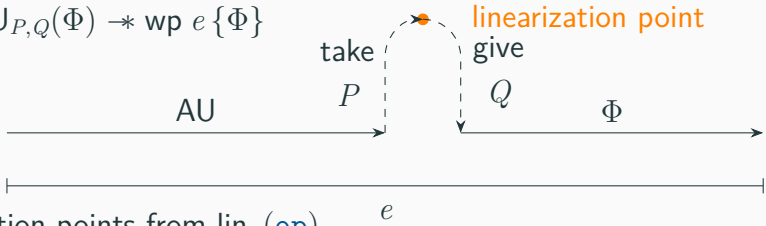

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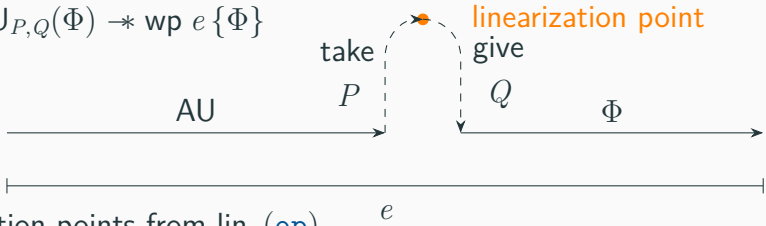

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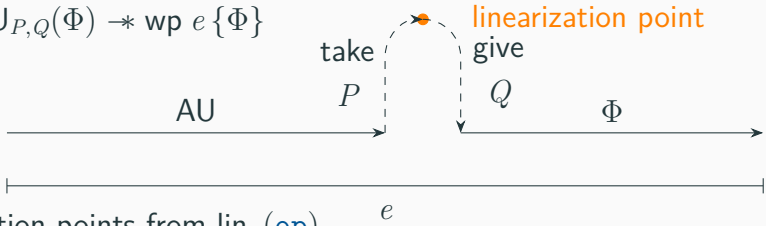

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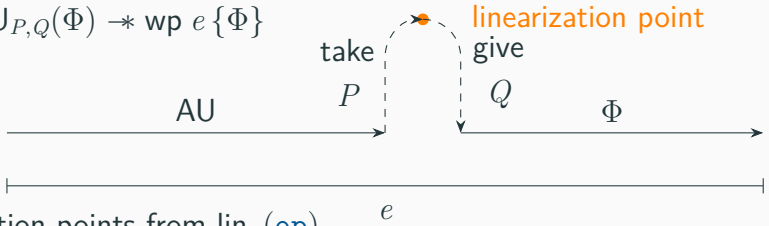

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For a library op, $\mathsf{lin}_\mu(\mathtt{op}) \implies \forall obj, x.\ I_\mu \vdash \langle P_\mu \rangle\ \mathtt{op}(obj, x)\ \langle y.\ Q_\mu(y) \rangle$

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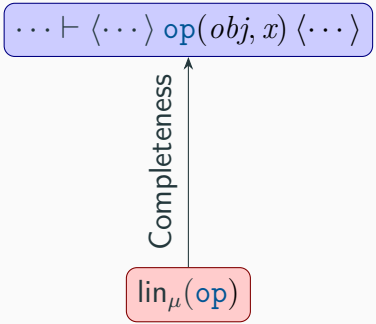
- Theoretically interesting



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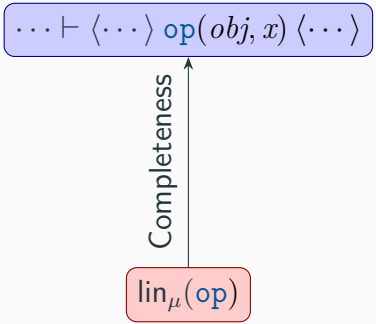

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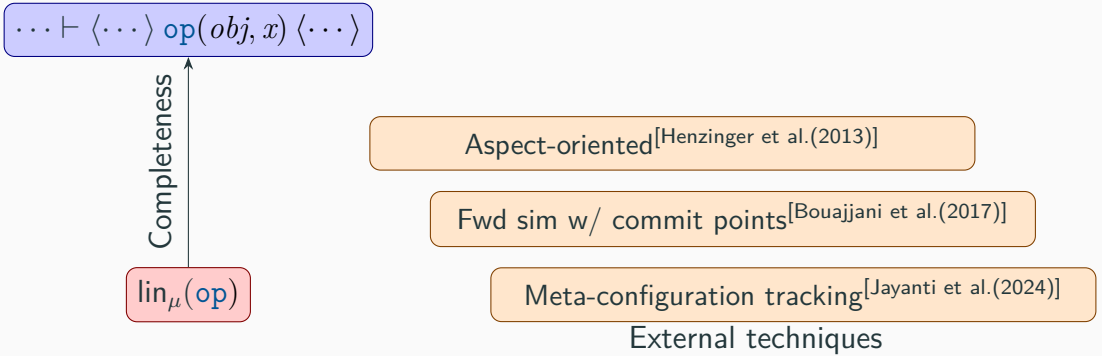

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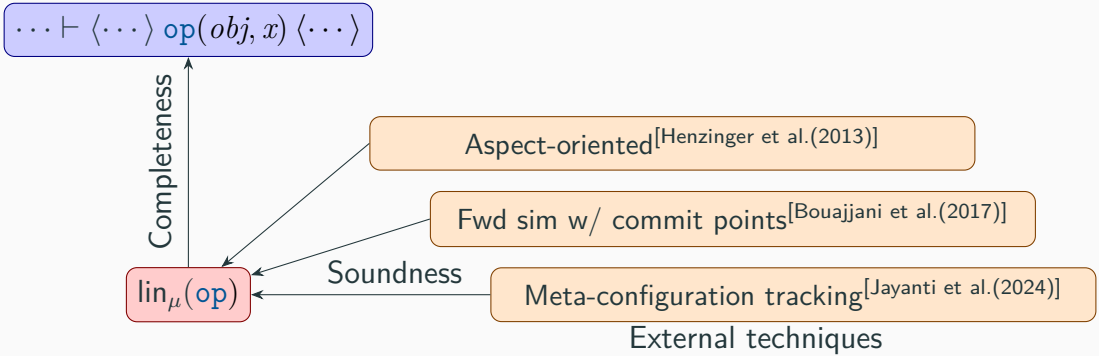

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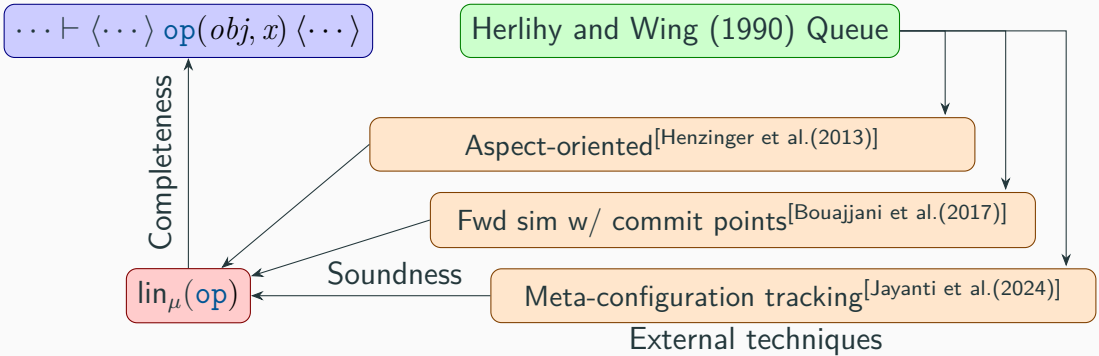

Conclusion

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- Language-independent whole program completeness theorems
- Case studies: HeapLang, λ_{Rust}, time credits, and Eris

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- Completeness of logically atomic triples
- Ported external techniques: aspect-oriented proofs, forward simulation with commit points, and meta-configuration tracking

Conclusion

Part 1. (To appear at ICFP 2026)

- Language-independent whole program completeness theorems
- Case studies: HeapLang, λ_{Rust}, time credits, and Eris

Please try it out!

(and/or join our discussion session)

`https://plf.inf.ethz.ch/research/icfp26-completeness.html`

Part 2.

- Completeness of logically atomic triples
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